the error Π_v is within the error of the direct problem. The results of the numerical experiments show that the error of the determination of combustion front velocity is only slightly greater than the error of the initial data — particularly in regard to the model parameters ρc, λ, z_{in}, z_{ex}, T, α — and that it increases somewhat with an increase in v.

NOTATION

s, width of the combustion front; v, velocity of the combustion front; τ, time; c, heat capacity; ρ, density; λ, thermal conductivity; T, temperature on the combustion front in the resevoir; τ_{max}, time of attainment of the maximum temperature at a given point of the region; Π_τ, error of measurement of τ_{max}; Π_v, error of determination of velocity v; Pe, Peclet number; Nu, Nusselt number; Fo, Fourier number.

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OPTIMUM PLANNING OF EXPERIMENTS IN THE IDENTIFICATION OF HEAT-TRANSFER PROCESSES

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An analysis is made of problems involving the optimum planning of nonsteady-state experiments conducted to identify thermal processes in structural materials and elements.

In mathematical models used for the theoretical analysis of the thermal operating conditions of different materials and structures, it is possible to distinguish three interconnected parts: 1) internal heat transfer; 2) heat transfer on the surface interacting with the environment; 3) applied thermal loads. Each of these components of the overall model is usually written approximately with allowance for the main governing factors, and each usually contains several characteristics. Identification methods, based on the solution of inverse heat-conduction problems, have recently begun to be widely used to determine these characteristics.

As an example, we will examine a unidimensional process involving the unilateral heating of a structural element with allowance for radiation from the heated surface. The mathematical model of the process has the form

$$C(T)\frac{\partial T}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial T}{\partial x}\right) + S(T),\ 0 < x < b,\ 0 < \tau \leqslant \tau_m, \tag{1}$$

$$T(x,\ 0) = T_0(x),\quad 0 \leqslant x \leqslant b, \tag{2}$$

$$\frac{\partial T(0,\ \tau)}{\partial x} = 0, \tag{3}$$

$$q_\lambda(\tau) = -\lambda(T(b,\ \tau))\frac{\partial T(b,\ \tau)}{\partial x} = q(\tau) - \varepsilon(T)\sigma T_w^4. \tag{4}$$

Equation (1) describes internal heat transfer in the material of the structural element and contains the characteristics C(T), λ(T), and S(T). Heat balance equation (4) establishes the model of heat transfer on the surface of the structure which interacts with the environment, and it includes the characteristic ε(T) and the thermal load $q(\tau)$. Here, the value of $q(\tau)$ can be determined by calculation [2]. The characteristics C(T), λ(T),

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S(T), and $\varepsilon(T)$ may be known with a very low accuracy or may be completely unknown. In such a case, the problem arises of identifying them, with a known thermal load $q(\tau)$, from the results of measurement of temperature as a function of the thrmal state inside the structure

$$T(X_i, \tau) = f_i(\tau), \quad i = \overline{1, N}, \tag{5}$$

where N is the number of measurement points.

In the general case, there is no unique solution for the entire set of characteristics from the solution of inverse problem (1)-(5). We must therefore resort to decomposing the thermal system being analyzed into discrete subsystems. In particular, we first need to determine the characteristics of internal heat transfer C(T), $\lambda(T)$, S(T) [3]. Here, model (4), describing the thermal interaction of the structure with the environment, is excluded from consideration as a result of the measurement of a boundary condition on the boundary x = b. This boundary condition might be of the second type, for example

$$-\lambda(T(b, \tau))\frac{\partial T(b, \tau)}{\partial x} = q(\tau), \tag{6}$$

where $q(\tau)$ is the thermal load on a subsystem in which internal heat transfer is taking place.

We then determine the characteristic $\varepsilon(T)$ with known characteristics C(T), $\lambda(T)$, and S(T) [4]. More complex phenomena [5] may take place on the heated surface, and in this case it will be necessary to determine several characteristics [6].

The inverse heat-transfer problem can be represented in the form of the operator equation

$$Au = f, \tag{7}$$

where A is a nonlinear operator constructed on the basis of the heat-transfer model being analyzed; u is the vector of the unknown charactersitics; $f = \{f_i(\tau)\}_1^N$ is the vector function of the measurements.

The model of state and, thus, the operator A depend on several quantities which determine the conditions of the experiments. These quantities include the geometric parameter b, the time of the experiment τ_m, the initial temperature distribution $T_0(x)$, and the thermal load $q(\tau)$. We combine these conditions into the vector $w = \{b, \tau_m, T_0(x), q(\tau)\}$. The operator A also depends on the scheme of temperature measurement $\xi = \{N, X\}$, where $X = \{X_i\}_1^N$ is the vector of the coordinates of the location of the thermocouples. The vectors w and ξ together comprise a factorial experiment

$$\pi = \{w, \xi\}. \tag{8}$$

An important problem in identifying heat-transfer characteristics is optimum experiment planning, which amounts to selecting a factorial experiment (8) that will ensure maximal accuracy in the determination of the characteristics of the process being examined [7]. The search for the optimum factorial experiment leads to the solution of the extremal problem

$$\pi_0 = \operatorname{Arg}\max_{\pi\in\Pi} \Psi(\pi), \tag{9}$$

where $\Psi(\pi)$ is a criterion of the quantity of the experiment, characterizing the accuracy of the solution of the inverse problem; Π is the set of the possible plans.

In solving inverse problems involving determination of temperature-dependent characteristics, it is customary to parameterize the sought functions with the use of a prescribed system of basis functions, such as B-splines [8]. As a result, the inverse problem is reduced to the determination of the vector of the coefficients of an approximation p of a certain dimensionality m. In this case, the criterion of the quality of the experiment is constructed on the basis of the information matrix [7]

$$M(\pi) = \{\Phi_{jk}, \quad j, k = \overline{1, m}\}, \tag{10}$$

where

$$\Phi_{jk} = \frac{1}{N}\sum_{i=1}^{N}\int_0^{\tau_m} \varkappa_i(\tau)\,\vartheta_j(X_i,\ \tau)\,\vartheta_k(X_i,\ \tau)\,d\tau;$$

$\kappa_i(\tau)$, $i = \overline{1, N}$ are functions accounting for information on the measurement errors; $\vartheta_k(x, \tau)$, $k = \overline{1, m}$ are sensitivity functions. The set of possible factorial experiments should consider the restrictions imposed by the use of specific experimental equipment in the realization of temperature experiments. For example, in seeking to find the optimum thermal load within a prescribed range $q \in Q$, it is often necessary to consider the restrictions

$$\alpha_1 \leqslant q(\tau) \leqslant \beta_1, \quad \alpha_2 \leqslant dq/d\tau \leqslant \beta_2, \tag{11}$$

which determine the energy capabilities of the experimental unit. Other restrictions can also be formulated.

In the general case, extremal problem (9), involving determination of all components of the factorial experiment (8), does not have a unique solution. Investigators usually analyze measurement-planning problems individually and select optimum boundary and initial conditions [9, 10]. Combined formulations of factorial experiments are also examined [11, 12]. Here, due to the nonlinearity of the inverse problem (7), it may be possible only to search for locally-optimum factorial experiments whose construction requires the assignment of a priori information on the characteristics being identified [13].

Practical use of factorial experiments requires analysis of several questions related both to the formulation of an extremal problem of type (9) and the development of algorithms for its numerical solution. First of all, it is necessary to substantiate the approach taken toward designation of the criterion of the quality of the experiment. Unfortunately, none of the criteria used [7] in the theory of factorial experimentation fully reflects the specific features of inverse heat transfer problems. It is thus necessary to prove the fitness of the chosen criterion, such as through computational experiments which simulate the identification procedure. Such an analysis was conducted in [14].

Formulation of the extremal problem in a factorial experiment requires careful analysis of all experimental conditions and measurements in order to isolate those which turn out to have a significant effect on the chosen quality criterion. Computational experiments can also be used for this purpose. We will examine this problem using the example of the problem of identifying the integral emissivity $\varepsilon(T)$ from conditions (1)-(5). The algorithm for the solution of the inverse problem for this case follows from [4]. The same study presented the initial data we will use.

Determination of the vector of the coefficients $\varepsilon = \{\varepsilon_k\}_1^m$, approximating the characteristic $\varepsilon(T)$ of the cubic B-spline, reduces to the solution of the system of equations

$$H\varepsilon = d, \tag{12}$$

where

$$H = \{h_{jk},\ j,\ k = \overline{1,\ m}\};$$

$$h_{jk} = \int_0^{\tau_m} \sigma^2 T_w^8\, \varphi_j(T_w)\, \varphi_k(T_w)\, d\tau;$$

$$d_k = \int_0^{\tau_m} (q_\lambda(\tau) - q(\tau))\,\sigma T_w^4\, \varphi_k(T_w)\, d\tau;$$

where σ is the Stefan–Boltzmann constant; $T_w(\tau)$ and $q_\lambda(\tau)$ are the temperature of the surface and the incident heat flux, determined from the solution of the inverse boundary-value problem [15]; $\varphi_k(T_w)$, $k = \overline{1, m}$ are basis B-splines. The matrix H is the information matrix of the system, while det H can serve as the criterion of the quality of the experiment [16]. We performed calculations in which we evaluated the effect of elements of the experiment on the criterion when they were varied according to the law

$$\tilde{\alpha} = k_\alpha \alpha,\ k_\alpha \in [0{,}5;\ 1{,}5],$$

where individual elements of the experiment were alternately examined as α. The results of the calculations are shown in Fig. 1 and illustrate that the quantities $q(\tau)$, τ_m, and b have

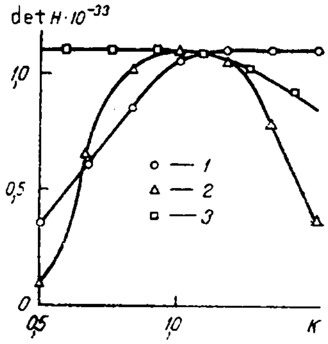

Fig. 1. Dependence of the quality criterion (det H) on the experimental conditions: 1) $q(\tau)$; 2) τ_m; 3) b.

a significant effect on the values of the criterion. At the same time, the initial temperature distribution $T_0(x)$ has almost no effect on the criterion and should be excluded from the experiment.

The solution of factorial thermal experiments entails the numerical solution of complex extremal problems of the type (9). In these problems, the quality criterion Ψ is constructed using the solutions of boundary-value problems for equations in partial derivatives (1)-(4), while restrictions such as (11) may be placed on the sought functions. To solve such problems, it is necessary to develop the corresponding algorithms and programs. Since the quality criteria of the experiment may not have convexity properties [7], then algorithms and programs should make use both of methods for searching for the global optimum of nonconvex functionals (such as the scanning method [17]) and gradient methods of optimization for convex quality criteria (such as the gradient projection method [18]).

Finally, there is a broad range of questions related to analysis and generalization of locally optimum experiments obtained in cases where the initial data is fairly indeterminate. This indeterminateness stems mainly from the need to assign a priori information on the characteristics being identified. It also has to do with the indeterminateness and relatively large errors in the reproduction of the thermal loads. If only locally optimum experiments can be conducted, then successive factorial experiments must be set up [13]. There have been almost no such investigations in regard to the identification of heat-transfer processes, and, in the interest of future generalizations, it would be most expedient to conduct these studies for individual classes of problems.

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IDENTIFICATION OF TWO-DIMENSIONAL HEAT FLOWS IN ANISOTROPIC BODIES OF COMPLEX FORM

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A method is proposed for numerical determination of two-dimensional temperature fields in anisotropic bodies with an arbitrary boundary for use in coefficient inverse heat-conduction problems.

In solving coefficient inverse heat-conduction problems, it is necessary to first evaluate the temperature field on the basis of approximate thermophysical characteristics (ATC). The availability of suitable methods and application packages makes it possible, by varying the approximately assigned ATC, to establish the empirical temperature fields that will be used in determining the sought ATC.

Here, we examine the formulation the numerical solution of two-dimensional nonlinear problems of heat conduction in anisotropic bodies in which complex heat transfer is taking place. Without simplications, the method makes it possible to identify full-scale temperature fields that are then used to determine the principal components λ_ξ, λ_η of the thermal conductivity tensor.

The mathematical model has the following form (Fig. 1):

$$c(T)\rho\frac{\partial T}{\partial \tau} = \operatorname{div}(\Lambda \operatorname{grad} T); \tag{1}$$

$$\left(\frac{\alpha}{c_p}\right)_{w1}(I_{e1} - I_{w1}) - \Lambda \operatorname{grad} T/_{w1} - \varepsilon_{w1}\sigma T_{w1}^4 = 0; \tag{2}$$

$$\alpha_{w2}(T_{e2} - T_{w2}) + \Lambda \operatorname{grad} T/_{w2} - \varepsilon_{w2}\sigma T_{w2}^4 = 0; \tag{3}$$

$$T(r,\ 0,\ \tau) = T_{w3}(r,\ \tau);\ T(r,\ \pi,\ \tau) = T_{w4}(r,\ \tau); \tag{4}$$

$$T(r,\ \theta,\ 0) = \varphi(r,\ \theta). \tag{5}$$

In curvilinear coordinates, the components of the thermal conductivity tensor have the form

$$\begin{aligned}\lambda_{rr} &= \lambda_\xi(T)\cos^2(\theta^\nu - \psi) + \lambda_\eta(T)\sin^2(\theta^\nu - \psi);\\ \lambda_{\theta\theta} &= \lambda_\xi(T)\sin^2(\theta^\nu - \psi) + \lambda_\eta(T)\cos^2(\theta^\nu - \psi);\\ \lambda_{r\theta} = \lambda_{\theta r} &= [\lambda_\eta(T) - \lambda_\xi(T)]\sin(\theta^\nu - \psi)\cos(\theta^\nu - \psi),\end{aligned} \tag{6}$$

where $\nu = 1$ for curvilinear coordinates and $\nu = 0$ for cartesian coordinates.

In solving boundary-value problem (1)-(5), we encounter the problem of allowing for the oblique derivative at the boundary w1 in boundary condition (2) and its relationship with the behavior of the boundary $r_{w1} = f(\theta)$.

After projecting the balance (2) in the direction of a normal to the boundary w1, the author of [1] obtained the following representation of the heat flux normal to the boundary w1:

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